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# Input pricing by an upstream monopolist into imperfectly competitive downstream markets*

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**Abstract**

In downstream markets where entry is independent from profitability conditions, the upstream supplier's optimal pricing policy is invariant with respect to downstream market structure. This price invariant result, however, is reversed when there is free entry in downstream market. When entry is endogenously dependent on profitability conditions, the upstream supplier's price-setting behavior depends on the number of operative firms in the final good market. We show that the upstream supplier charges a higher input price under a free entry situation in downstream market than under a no-entry condition. We also show that a higher input price is set under Bertrand competition than under Cournot competition in a downstream market with free entry.

*JEL Classification*: L11; L13; D43

*Keywords*: Vertical relationship; Pricing; Competition; Oligopoly; Free entry

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## 1. Introduction

A branch of literature that has received considerable attention examines the upstream supplier's optimal pricing policy with respect to downstream market competition. Greenhut and Ohta (1976) and Tyagi (1999) concluded that the price-setting behavior of an upstream input supplier does not depend on the number of downstream firms for the constant elasticity of slope demand function.[1] Both papers consider the case of an exogenous market structure.

In this paper, we show that this price invariance result is reversed when there is free entry in the downstream market. We consider the case where products are differentiated and demand structure is linear.

In this setup, the upstream input price (wholesale price) is sensitive to downstream market competition, and more specifically it depends on the number of operative firms (retailers) in the final good market. Hence, free entry condition in downstream market affects the upstream monopolist's optimal pricing policy and the price invariance result is no longer valid.

If the upstream supplier moves first by setting the price of an intermediate good anticipating free entry in the downstream market, she will use her first mover advantage in order to influence the degree of *competition for the market*. This effect cannot be present in models which consider a fixed number of downstream firms, and therefore the upstream monopolist can only influence the intensity of *competition in the market*.

In a no-entry situation, the number of firms enters as a multiplicative factor, both for the downstream firm's profit and for the total revenue of the upstream monopolist. The neutrality result follows. In the case of free entry, however, the number of firms depends on the *level* of profit of downstream firms generated. Hence, with a higher input price, there is the previous effect (influencing competition in the market), and another one through the number of firms given the presence of elasticity of the number of firms with respect to the input price.

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[1] Consider a homogenous-good market with inverse demand function $p(Q)$, where $Q$ is the industry output and $p'(Q) < 0$ when $p(Q) > 0$. The elasticity of slope of inverse demand function is $p''(Q)Q/p'(Q)$. See Tyagi (1999) for a detailed discussion and a list of specific demand functions that exhibit constant elasticity of slope.

The effect through the number of firms is always negative; by lowering the price of the input the upstream monopolist induces more entry. Consequently, a lower input price is set under a free entry situation in downstream market than under a no-entry condition.

The above conclusions for the upstream monopolist's input supply price under free entry hold for both quantity and price setting behavior of downstream firms. For the Cournot case a full analytical proof is provided, while for the Bertrand case extensive numerical analysis is used.

Numerical analysis is also used to determine whether under free entry, the upstream monopolist charges a higher input price under quantity or price competition in the downstream market. When products become more substitutable, the upstream monopolist finds the distortion on total quantity under price setting behavior of downstream rivals less important than under quantity setting behavior. Therefore, a higher input price is set under free entry Bertrand competition than under free entry Cournot competition.

Cellini *et al*. (2004) and Mukherjee (2005) carry out comparative welfare evaluations between price and quantity competition on oligopolistic markets with free entry. These contributions point out that, with an endogenous market structure, welfare is higher under Bertrand competition when products are close substitutes, and it is higher under Cournot competition for sufficiently differentiated products. We show that these standard welfare conclusions emerge if retailers buy inputs from an independent upstream supplier.

Haring & Kaserman (1978) and Greenhut & Ohta (1978) show that, for a given number of firms, the price invariance result does not hold when increasing marginal costs are considered. Koulamas and Kyparisis (2009) consider the effects of entry on downstream operational efficiency and demonstrate that the wholesaler's pricing policy is affected when pre-entry and post-entry retailer's variable costs differ. Our analysis extends this literature by showing that the price invariant result does not hold when free entry is taken into account, even if constant marginal costs are assumed and the effects of entry on downstream operational efficiency are ignored. Moreover, none of these studies consider retail price competition.

The rest of this paper is organized as follows. Section 2 presents the key elements of the baseline model. Section 3 briefly discusses the case where the number of downstream rivals is fixed. Section 4 considers both Cournot and Bertrand free entry downstream equilibrium and provides the results. The final section concludes.

## 2. The Baseline Model

Consider an economy with two final goods, X and M; the latter is a homogeneous *numeraire good* produced by a competitive sector while product X is sold in an imperfectly competitive market. Assume an upstream monopolist, which is the provider of an essential input for the downstream production of final good X. One unit of retail output requires one unit of the input. The monopolist charges a price $d$ for the input and the upstream marginal cost is set equal to zero. Let $n$ be the number of retail competitors in the monopolistic sector, each producing a variety of differentiated good. Each downstream firm produces a single product and each product is produced by only one firm. All firms face identical cost functions, composed of a fixed cost $f$ and a constant variable cost. For simplicity, we assume that the marginal cost of production for a downstream firm is the price $d$ of the intermediate input supplied by the upstream monopolist.

Let $m$ be the quantity of the outside good which is assumed to be produced at a constant marginal cost equal to 1, and that its competitive price is 1. The utility function is additively separable in $m$ and therefore there are no income effects on the monopolistic sector; this enables us to perform partial equilibrium analysis. Following Bowley (1924), consumer preferences are represented by a utility function of the general form: [2]

$$U = a\sum_{i=1}^{n} q_i - \frac{1}{2} b\sum_{i=1}^{n} q_i^2 - \theta b\sum_{i=1}^{n} q_i \sum_{i\neq j}^{n-1} q_j + m \qquad (1)$$

The simplifying assumption that the slope parameter ($b$) is equal to 1 is adopted. Consumer demand for the retail product of firm $i$ is given by the inverse demand function:

[2] In Bowley's formulation, market size increases as the number of varieties increases. The Bowley model specification is also used by Spence (1976) and Dixit (1979).

$$p_i = a - q_i - \theta \sum_{i \neq j}^{n-1} q_j \tag{2}$$

where $p_i$ is the price of firm $i$'s product, $q_{i,}q_j$ are the outputs of firm $i$ and $j$ respectively $(i, j = 1, 2 \ldots n, i \neq j)$, and $a$ is a strictly positive constant. The parameter $\theta \in [0,1]$ shows the degree of product differentiation. As $\theta$ approaches 0, the products of retail rivals become independent. As $\theta$ approaches 1, the products of firms become closer substitutes. In the extreme case of $\theta = 1$ products are completely homogeneous.

The equilibrium outcomes are derived using backward induction. First, the firms' decision variables under different forms of retail competition are determined, and then the upstream monopolist maximizes its profits, subject to the equilibrium demand for its output under each form of retail competition. The types of retail competition examined are Cournot oligopoly, Bertrand oligopoly and oligopoly with free entry, *i.e.*, the number of downstream firms, under quantity and price setting behavior, is determined by the zero profit condition.

## 3. Fixed number of downstream firms

Initially, the case where the number of firms is exogenously determined is briefly discussed. There is prevention of new entry and the number of retailers is fixed at $n = \bar{n}$. Under quantity competition, firms act simultaneously and choose output levels independently. The Cournot market equilibrium is derived straightforwardly when profits are non-negative:

$$q^c = \frac{a - d}{[2 + (n-1)\theta]} \tag{3}$$

$$p^c = d + \frac{a - d}{[2 + (n-1)\theta]} \tag{4}$$

In the Bertrand model, firms simultaneously set prices rather than output. By inverting equation (2), we obtain the demand function implied by the Bowley model:

$$q_i = \frac{(1-\theta)(a-d) - [1+\theta(n-2)](p_i - d) + \theta \sum_{i \neq j} (p_j - d)}{(1-\theta)[1+\theta(n-1)]} \tag{5}$$

which is valid provided that $\theta < 1$.[3] Standard analysis yields the following Bertrand market equilibrium:

$$q^b = \frac{[1+(n-2)\theta](a-d)}{[1+(n-1)\theta][2+(n-3)\theta]} \tag{6}$$

$$p^b = d + \frac{(1-\theta)(a-d)}{[2+(n-3)\theta]}, \tag{7}$$

Eqs. (3) to (7) are in line with Cellini *et al*. (2004), Mukherjee (2005) and Koh (2008).

Under vertical separation, the independent upstream supplier sets profit maximizing input prices. With an exogenous market structure, she will charge an input price independently of the number of downstream firms and/or the type of competition, as Proposition 1 states (See Appendix A for proof). A hat is used to denote the input price when the number of retailers is fixed.

**Proposition 1.** *Under an exogenous downstream market structure, the upstream monopolist sets its input price equal to* $\hat{d} = a/2$ *under both Cournot and Bertrand competition.*

Proposition 1 corresponds to Greenhut and Ohta's (1976) price invariance result. The profit maximizing input price charged by the upstream monopolist does not depend on the number of retailers in the final good market. Proposition 1 shows that the upstream monopolist charges the same input price regardless of whether competition in the final good market is of the Cournot or Bertrand type. Moreover, it reveals that this result holds not only for the case of homogenous final products ($\theta = 1$) but also for the case of differentiated final goods ($0 \leq \theta < 1$), that is regardless of the degree of product substitutability.

[3] Without loss of generality, prices are expressed as deviations from marginal cost because it is constant for each firm.

Arya *et al* (2008) reach a similar conclusion, considering a classic model of duopoly competition and assuming that firms have different marginal costs. In the present setting, imposing more symmetry on the model, we show that the upstream supplier charges the same input price under Cournot and Bertrand competition for any alternative downstream market structure, that is for any $n \geq 1$.

Singh & Vives (1984), considering a duopoly setting and assuming that firms have constant yet different marginal costs, show that consumer and total surplus are always higher in the Bertrand equilibrium while firms profits are higher in the Cournot equilibrium when goods are substitutes. By imposing more symmetry, Koh (2008) shows that duopoly welfare results generalize to the *n*-firm setting.[4] Proposition 1 implies that the cost structure of retail providers is not affected by the form of retail competition. Therefore, the welfare conclusions, initially proposed by Singh & Vives (1984) and later generalized by Koh (2008), prevail even if retail competitors buy inputs from an upstream monopolist.

## 4. Free entry downstream equilibrium

In this section, we turn to the case where the number of firms in the downstream market is endogenously determined. The number of retailers is considered to be continuous, suggesting a zero profit condition. We shall consider both Cournot and Bertrand competition.

### *4.1 Cournot downstream competition*

In a quantity setting behavior, the zero profit condition is expressed by

$$\frac{(a-d^c)^2}{[2+\theta(n^c-1)]^2} = f \, , \tag{8}$$

for which the long run equilibrium number of firms is obtained as

[4] See Hackner (2000) for the case where in the presence of more than two firms producing vertically differentiated goods, the duopoly results proposed by Singh and Vives (1984) do *not* generalise to the *n*-firm setting.

$$n^c(d^c) = \frac{a - d^c}{\theta\sqrt{f}} - \frac{2-\theta}{\theta} \tag{9}$$

We assume that the following condition holds throughout the analysis

$$(a - d^c)^2 \geq 4f \;, \tag{A}$$

which ensures that at least one firm enters the market under free entry equilibrium. Taking the derivative of expression (9) with respect to $\theta$ we have

$$\frac{\partial n^c(d^c)}{\partial\theta} = \frac{2\sqrt{f} - (a - d^c)}{\theta^2\sqrt{f}} < 0\,, \tag{10}$$

for $(a - d^c) > 2\sqrt{f}$ , which is always true given (A). Thus, the higher is $\theta$ (*i.e.*, the more substitutable products are) the smaller is the number of firms that enter the market.

Substituting Eq. (9) into Eqs. (3) and (4), we obtain the final equilibrium quantity and prices:

$$p^c = d^c + \sqrt{f} \tag{11}$$

$$q^c = \sqrt{f} \tag{12}$$

Eqs. (8) to (12) are in line with Mukherjee (2005).The upstream monopolist chooses the input price *d* in order to maximize the following profit function

$$\Pi = d^c \; n^c(d^c) \; q^c \tag{13}$$

A tilde is used to denote the input price charged by the upstream monopolist when there is free entry in the downstream market. The profit maximizing input price $\tilde{d}^c$ is given by

$$\tilde{d}^c = \frac{a}{2} - \frac{2-\theta}{2}\sqrt{f} \qquad (14)$$

Equation (14) implies that the profit maximizing input price depends on the degree of product substitutability $\theta$ and the fixed set-up cost $f$, and therefore it is related to the number of retailers. Thus, the result of the supplier's optimal pricing policy being invariant to downstream market structure is reversed if free entry is taken into account, as the next Proposition indicates.

**Proposition 2.** *Under Cournot competition in a downstream market with free entry, the optimal input price the upstream supplier sets is sensitive to downstream market structure, and it depends on the number of operative firms.*

In models which consider a fixed number of downstream firms, the upstream monopolist can only influence the intensity of *competition in the market*. The number of firms enters as a multiplicative factor, both for the downstream firm's profit and for the total revenue of the upstream monopolist. The neutrality result follows.

In the case of free entry, however, the upstream supplier not only can influence the intensity of *competition in the market* but also can influence the degree of competition in the downstream market, that is *competition for the market*. The number of firms depends on the *level* of profit of downstream firms generated. Hence, with a higher input price, there is the previous effect (influencing competition in the market), but there is another effect through the number of firms since there is now elasticity of the number of firms with respect to the input price.

The free entry equilibrium depends on the level of fixed costs and the degree of differentiation, and therefore the profit maximizing input price will also depend on these variables. Comparative statics show that the optimal input price the upstream supplier sets i) decreases with fixed set-up costs and ii) increases as products become more substitutes.

It is obvious from Eq. (14) that the higher is $f$ the lower is the optimal input price the upstream monopolist sets, for any given degree of product differentiation. When the downstream market is very concentrated, the benefits from increasing competition are important. As fixed cost $f$ increases, the monopolist finds the distortion on total quantity more important and thus charges a lower input price.

Moreover, the higher is $\theta$ the higher is the optimal input price the upstream monopolist sets, for any given fixed cost $f$. For any given market concentration $(\forall n)$, firm's conduct becomes more competitive as products become more substitutes. Thus, in turn, makes it less urgent for the upstream monopolist to increase competition through a lower input price and a larger number of downstream firms.

The independent upstream supplier sets a lower input price under a free entry situation in downstream market than under a no-entry condition, as proposition 3 states (See Appendix B for proof). Recall that a tilde is used to denote the input price charged by the upstream monopolist under free entry and a hat to denote the input price when the number of retailers is fixed.

**Proposition 3.** $\tilde{d}^c - \hat{d}^c < 0$ *for* $\forall(f;\theta)$ *with* $\lim_{f \to 0}(\tilde{d}^c - \hat{d}^c) = 0$

The only case where the two prices coincide is when $f$ approaches zero. For a given number of firms, the upstream monopolist cannot affect downstream market structure by changing the input price she sets. Under free entry, however, there is elasticity of the number of firms with respect to input price. This elasticity is always negative; a higher input price implies higher marginal cost for each retailer, and thus fewer firms in equilibrium. By lowering the price of the input the upstream supplier induces more entry.

*4.2 Bertrand downstream competition*

In a price setting behavior the zero profit condition is expressed by

$$\frac{(a-d^b)^2(1-\theta)[(1+\theta(n^b-2)]}{[1+\theta(n^b-1)][2+\theta(n^b-3)]^2} = f \qquad (15)$$

It is assumed that the following condition holds throughout the analysis

$$(a-d^b)^2 \geq 4f \;, \qquad (B)$$

which ensures that at least one firm enters the market under free entry equilibrium.

Solving Eq (15) for $n^b$, we take the expression for the long run equilibrium number of firms as a function of $d^b$, and then substituting $n^b(d^b)$ into Eqs. (6) and (7) we get the final equilibrium quantity and prices. The upstream monopolist chooses the input price $d$ in order to maximize the following profit function

$$\Pi = d^b \; n^b(d^b) \; q^b \tag{16}$$

However, since Eq. (15) is a polynomial equation of third degree in the variable $n$ with all its coefficients being non numeric, the exact formula for the equilibrium number of firms is too complex. Consequently, the exact formula for the profit maximizing input price $\tilde{d}^b$ is extremely difficult to be derived. In order to be able to determine whether the profit maximizing input price depends on the number of retailers or not, we proceed by using numerical examples.

In the calculations, the values of the parameters $a$, $\theta$ and $f$ vary. In Table I, $\tilde{d}^b$ and $\tilde{n}^b$ are the equilibrium input price and equilibrium number of firms, respectively, under free entry Bertrand competition in the downstream market, while $\tilde{d}^c$ and $\tilde{n}^c$ are the equilibrium input price and equilibrium number of firms, respectively, under free entry Cournot competition. The last column reports the profit maximizing input price when the number of downstream firms is fixed. Recall from Proposition 1 that the upstream monopolist charges the same input price under either Bertrand or Cournot competition (i.e. $\hat{d}^c = \hat{d}^b$), so $\hat{d}$ does not bear a superscript. There are nine groups of calculations, distinguished by the degree of product substitutability. Within each group, fixed costs increase.

The calculations reported in Table I show that Proposition 2 remains valid even for Bertrand retail competition. The profit maximizing input price charged under price setting behavior of downstream rivals depends on the fixed cost $f$ and the degree of product differentiation $\theta$, and thus it is related to the number of retailers.[5] Moreover, the independent upstream supplier sets a lower input price when the number of downstream firms is endogenously determined compared to the exogenous case.

(Place Table I _ approximately here)

[5] Exactly like in the Cournot case, the higher is the fixed cost $f$ and the less substitutable products are, the lower is the profit maximizing input price that the upstream monopolist sets.

*4.3 Comparison between Cournot and Bertrand downstream competition*

The calculations reported in Table I illustrate a number of points concerning the comparison between Cournot and Bertrand equilibrium outcomes in the presence of an upstream monopolist.

First*, with free entry in the downstream market, the upstream monopolist charges a higher input price under price competition than under quantity competition.* The lower is $\theta$ (the less substitutable products are) the smaller is the positive difference $\tilde{d}^b - \tilde{d}^c$. In Table I, for very small $\theta$ (see Groups 8 and 9), the prices charged under Cournot and Bertrand free entry equilibrium are almost equal. In the extreme case of totally independent products $(\theta = 0)$, the prices charged under the two regimes are equal, *i.e.*, $\tilde{d}^b = \tilde{d}^c$. [6] As products become more substitutable, the upstream monopolist finds the distortion on total quantity under price setting behavior of downstream rivals less important than under quantity setting behavior.

Second, *the equilibrium number of downstream firms competing in prices* $(\tilde{n}^b)$ *is not larger than the equilibrium number of downstream firms competing in quantities* $(\tilde{n}^c)$. Under free entry, holding the cost structure of retail rivals constant, the long-run equilibrium number of firms will be larger under quantity setting than under price setting behavior (Cellini *et al*, 2004; Mukherjee 2005). Since the upstream supplier charges a higher input price under retail price competition, the marginal cost of a retailer competing in prices will be higher than the marginal cost of the same retailer competing in quantities. Therefore, Bertrand firms are still fewer in equilibrium.[7]

Cellini *et al*. (2004) and Mukherjee (2005) investigate the welfare properties of the entry process in oligopolistic markets. They find that Bertrand competition yield a higher welfare than Cournot competition if products are close substitutes and a lower

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[6] The equilibrium input price and the equilibrium number of downstream competitors must satisfy the zero profit condition. Substituting $\tilde{d}^c, \tilde{n}^c$ and $\tilde{d}^b, \tilde{n}^b$ into expressions (8) and (15) respectively, and combining these expressions together we obtain:

$$(a-\tilde{d}^c)^2(1+\theta(\tilde{n}^b-1))(2+\theta(\tilde{n}^b-3))^2 = (a-\tilde{d}^b)^2(1-\theta)(1+\theta(\tilde{n}^b-2))(2+\theta(\tilde{n}^c-1))^2 .$$

It is easy to verify that for $\theta = 0$, the above equation holds for $\tilde{d}^c = \tilde{d}^b$.

[7] Although the number of retailers is assumed to be continuous in an attempt to calculate the profit maximizing input price under both Cournot and Bertrand free entry equilibrium, I present the equilibrium values of $\tilde{n}^b$ and $\tilde{n}^c$ as integers. This explains why there are cases in Table I where $\tilde{n}^b$ and $\tilde{n}^c$ are equal.

welfare if products are sufficiently differentiated. We show that these welfare conclusions are not altered if retail competitors buy inputs from an upstream supplier.

Ignoring the integer constraint on number of firms implies that the net profit of all downstream firms is zero in the free entry equilibrium. Therefore, total welfare is equal to consumer surplus, which is given by

$$CS = \frac{(a-d)q}{2} = \frac{nq^2(1+\theta(n-1))}{2} \tag{17}$$

All subsequent equations base on Mukherjee (2005). The optimal output under Cournot competition is given by Eq. (12). From Eqs. (6) and (15) we find that the optimal output under Bertrand competition as a function of input price $d^b$ and number of firms $n^b$ is

$$q^b = \frac{f(2+\theta(n^b-3))}{(a-d^b)(1-\theta)} \tag{18}$$

Consumer surplus under Cournot and Bertrand competition is respectively

$$CS^c = \frac{n^c f(1+\theta(n^c-1))}{2} \tag{19}$$

$$CS^b = \frac{n^b f^2[1+\theta(n^b-1)][2+\theta(n^b-3)]^2}{2(a-d^b)^2(1-\theta)^2} \tag{20}$$

From Eqs. (19) and (20) it is clear that $CS^c (>=<) CS^b$ if and only if

$$n^c(1+\theta(n^c-1))(a-d^b)^2(1-\theta)^2 (>=<) n^b f[1+\theta(n^b-1)][2+\theta(n^b-3)]^2 \tag{21}$$

For $\theta$=1 the LHS of (21) is less than RHS. Expression (21) becomes $0<(n^b)^2(n^b-1)f$ , which is true given that assumption (B) ensures that $n^b>1$. However, for $\theta$=0 the RHS is now greater than LHS. Expression (21) becomes $n^c(a-d^b)^2>4fn^b$ , which is true since $(a-d^b)^2>4f$ (see assumption B) and $n^c \geq n^b$ (see Table I).

Welfare is higher under Bertrand competition if products are close substitutes and it is higher under Cournot competition for sufficiently differentiated products. Therefore, the standard welfare conclusions emerge if retailers buy inputs from an independent upstream supplier.

## 5. Conclusions

We considered the case of an upstream monopolist producing the intermediate input and an imperfectly competitive downstream stage, with retailers producing the final differentiated good. Our model showed that the result of the supplier’s optimal pricing policy being invariant to downstream market structure is reversed when there is free entry. The upstream input price is sensitive to downstream market competition, and more specifically it depends on the number of downstream firms. Free entry condition in downstream market affects optimal upstream pricing and the price invariance result obtained under no-entry condition no longer holds.

We showed that the upstream supplier charges a lower input price when the number of downstream firms is endogenously determined (free entry) compared to the case when the latter is determined exogenously (no-entry condition), for both quantity and price retail competition. We also showed that a higher input price is set under Bertrand competition than under Cournot competition in a downstream market with free entry. Furthermore, the standard welfare results of the previous literature comparing Bertrand and Cournot competition under free entry emerge if retail competitors procure inputs from an upstream supplier.

Future research might consider alternative demand formulations, different cost structures and upstream potential competition. Although these extensions may provide new insights of interest, they seem unlikely to reverse the finding that in a downstream market with free entry the upstream input price depends on the number of operative firms.

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## APPENDIX A

**Proof of Proposition 1.**

*i) Cournot competition*

The profit function for a downstream firm can be expressed as

$$\Pi_i^c = p_i q_i - dq_i - f = (a - q_i - \theta \sum_{i \neq j}^{n-1} q_j) q_i - dq_i - f \tag{A.1}$$

The Cournot downstream equilibrium can be defined by maximizing the above profit function with respect to $q_i$ and imposing symmetry by setting $q_i = q$ for all i. Given the number of downstream firms *n*, the symmetric equilibrium output per firm is

$$q^c = \frac{a-d}{2+(n-1)\theta}$$

which is Eq. (3). The upstream monopolist maximizes its profits with respect to input price *d*, subject to the Cournot equilibrium in the downstream stage. Its profit function can be expressed as

$$\Pi^U = d^c \ q^c \ \overline{n} = d^c \frac{(a-d^c)}{[2+\theta(\overline{n}-1)]} \overline{n} \tag{A.2}$$

Taking the first order condition

$$\frac{\partial \Pi^U}{\partial d^c} = 0 \Rightarrow \ \overline{n}a - 2\overline{n}d^c = 0$$

and solving for $d^c$ we have

$$\hat{d}^c = \frac{a}{2} \tag{A.3}$$

*ii.) Bertrand competition*

When firms compete in prices, the profit function of firm *i* can be expressed as

$$\Pi_i^b = (p_i - d)\frac{(1-\theta)(a-d) - [1+\theta(n-2)](p_i - d) + \theta\sum_{i\neq j}(p_j - d)}{(1-\theta)[1+\theta(n-1)]} - f \tag{A.4}$$

Maximizing (4) with respect to $p_i$ , imposing symmetry by setting $p_i = p$ for all i and solving for p, we get Eq. (6) and (7). Subject to this symmetric equilibrium, the wholesale supplier maximizes

$$\Pi^U = d^b\ q^b\ \overline{n} = d^b\ \frac{[1+(\overline{n}-2)\theta](a-d^b)}{[1+(\overline{n}-1)\theta][2+(\overline{n}-3)\theta]}\overline{n} \tag{A.5}$$

with respect to input price $d^b$ . From the first order condition we get

$$\frac{\partial \Pi^U}{\partial d^b} = 0 \Rightarrow\ \overline{n}a[1+\theta(\overline{n}-2)] - 2\overline{n}d^b[1+\theta(\overline{n}-2)] = 0$$

and solving for $d^b$ we have

$$\hat{d}^b = \frac{a}{2} \tag{A.6}$$

From (A.3) and (A.6) it is obvious that $\hat{d}^c = \hat{d}^b$ .

APPENDIX B

**Proof of Proposition 3.**

*Cournot free entry equilibrium*

The upstream monopolist chooses the input price $d^c$ in order to maximize the following profit function

$$\Pi = d^c \ n^c(d^c) \ q^c$$

which is Eq. (13). Substituting equilibrium output $q^c$ (Eq. (12)) and equilibrium number of firms $n^c(d^c)$ (Eq. (9)) into (13) we obtain

$$\Pi^U = d^c[\frac{a-d^c}{\theta\sqrt{f}} - \frac{2-\theta}{\theta}]\sqrt{f} \tag{B.1}$$

From the first order condition we get

$$\frac{\partial \Pi}{\partial d^c} = 0 \Rightarrow \tilde{d}^c = \frac{a}{2} - \frac{2-\theta}{2}\sqrt{f}$$

which is Eq. (14). A direct comparison between (14) and (A.3) reveals that the independent upstream supplier sets a lower input price when the number of downstream firms is endogenously determined than when the number of downstream firms is exogenously determined.

Table I.
EQUILIBRIUM INPUT PRICE AND NUMBER OF FIRMS UNDER COURNOT AND BERTRAND COMPETITION

| | | | | Cournot free entry equilibrium | | Bertrand free entry equilibrium | | Fixed number of firms |
|---|---|---|---|---|---|---|---|---|
| GROUP | | | | $\tilde{d}^c$ | $\tilde{n}^c$ | $\tilde{d}^b$ | $\tilde{n}^b$ | $\hat{d}$ |
| 1 | $a=10$ | $\theta=0.9$ | $f=1$ | 4.450 | 4 | 4.646 | 2 | 5 |
| | | | $f=2$ | 4.222 | 3 | 4.449 | 1 | 5 |
| 2 | $a=10$ | $\theta=0.8$ | $f=1$ | 4.400 | 5 | 4.532 | 3 | 5 |
| | | | $f=2$ | 4.151 | 3 | 4.309 | 2 | 5 |
| | | | $f=5$ | 3.658 | 2 | 3.764 | 1 | 5 |
| 3 | $a=10$ | $\theta=0.7$ | $f=1$ | 4.350 | 6 | 4.439 | 3 | 5 |
| | | | $f=2$ | 4.080 | 4 | 4.188 | 2 | 5 |
| | | | $f=5$ | 3.546 | 2 | 3.629 | 1 | 5 |
| 4 | $a=10$ | $\theta=0.6$ | $f=1$ | 4.300 | 7 | 4.359 | 5 | 5 |
| | | | $f=2$ | 4.010 | 4 | 4.083 | 3 | 5 |
| | | | $f=5$ | 3.434 | 2 | 3.495 | 2 | 5 |
| 5 | $a=10$ | $\theta=0.5$ | $f=1$ | 4.250 | 8 | 4.288 | 6 | 5 |
| | | | $f=2$ | 3.939 | 5 | 3.986 | 4 | 5 |
| | | | $f=5$ | 3.322 | 2 | 3.365 | 2 | 5 |
| | | | $f=7$ | 3.015 | 2 | 3.039 | 2 | 5 |
| 6 | $a=10$ | $\theta=0.4$ | $f=1$ | 4.200 | 10 | 4.223 | 8 | 5 |
| | | | $f=2$ | 3.868 | 6 | 3.897 | 5 | 5 |
| | | | $f=5$ | 3.211 | 3 | 3.238 | 3 | 5 |
| | | | $f=7$ | 2.883 | 2 | 2.901 | 2 | 5 |
| 7 | $a=10$ | $\theta=0.3$ | $f=1$ | 4.150 | 13 | 4.162 | 12 | 5 |
| | | | $f=2$ | 3.798 | 8 | 3.813 | 7 | 5 |
| | | | $f=5$ | 3.099 | 4 | 3.114 | 4 | 5 |
| | | | $f=7$ | 2.751 | 3 | 2.763 | 3 | 5 |
| 8 | $a=10$ | $\theta=0.2$ | $f=1$ | 4.100 | 20 | 4.105 | 18 | 5 |
| | | | $f=2$ | 3.727 | 13 | 3.733 | 12 | 5 |
| | | | $f=5$ | 2.987 | 6 | 2.994 | 6 | 5 |
| | | | $f=7$ | 2.618 | 4 | 2.624 | 4 | 5 |
| | | | $f=10$ | 2.154 | 3 | 2.157 | 3 | 5 |
| 9 | $a=10$ | $\theta=0.1$ | $f=1$ | 4.050 | 40 | 4.051 | 38 | 5 |
| | | | $f=2$ | 3.656 | 25 | 3.658 | 24 | 5 |
| | | | $f=5$ | 2.876 | 12 | 2.877 | 12 | 5 |
| | | | $f=7$ | 2.486 | 9 | 2.488 | 9 | 5 |
| | | | $f=10$ | 1.995 | 6 | 1.997 | 6 | 5 |
| | | | $f=14$ | 1.445 | 3 | 1.446 | 3 | 5 |